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Visualizing the log-periodic pattern before crashes

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Abstract. We present a method for visualizing the pattern which we believe to be a precursor signature of financial crashes (or ruptures). The log-periodicity of the pattern is investigated through the envelope function technique. Three periods of the Dow Jones Industrial Average (DJIA) are investigated: 1982–1987, 1992–1997 and 1993–1998. The presence of a rupture in the end of 1998 is outlined from data taken before the end of August 1998.

PACS. 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 47.53.+n Fractals

1 About a financial crash occurence

The application of statistical physics ideas to the forecasting of stock market behavior has been proposed long ago [1] and pursued in the pioneer work of physicists interested by economy laws [2,3]. Econophysics [4] aims to fill the huge gap separating "empirical finance" and "econometric theories". Various subjects have been approached like the option pricing, stock market data analysis, market modelling and forecasting, etc. It should be also noted that economist authors (without the help of physicists) have already applied physics methods to market modelling [5]. Also, physicists have recently used techniques coming from the world of finance, like moving averages [6].

Among noticeable events, crashes are spectacular ones. Even though a stock market crash is considered as a highly unpredictable event, it should be noticed that it takes place systematically during a period of generalized anxiety spreading over the markets. The crash can be seen as a natural correction after euphory bringing the market to a "normal state". Two important facts should be underlined:

(i) the series of daily fluctuations of stock markets present a huge clustering around the crash date, *i.e.* huge fluctuations are grouped around the crash date. This is well illustrated in Figure 1 for the case of the Dow Jones Industrial Average (DJIA) around 1987. The time span of this clustering is quite long: a few years. This clustering indicates that larger and larger fluctuations take place before crashes;

(ii) a second remark concerns the panic-correlations appearing before crashes. This kind of collective behavior is commonly observed during a trading day. The market in Tokyo closes before London opens and thereafter New York opens. During periods of panic, financial analysts are looking for the results and evolution of the geographically preceding market. Strong correlations are found to be existing inbetween market fluctuations before crashes.

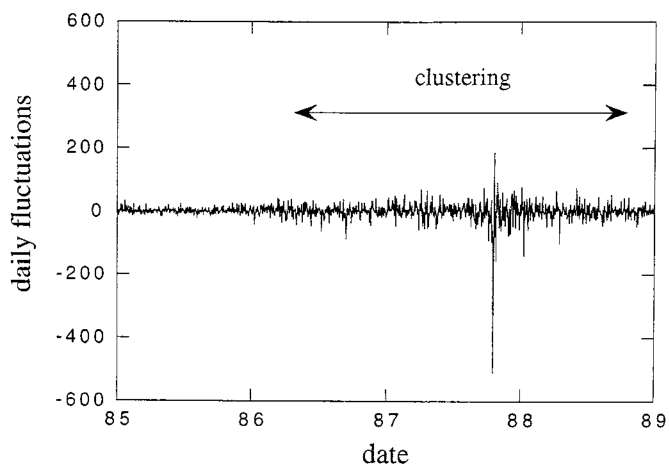

Fig. 1. The DJIA fluctuations before and around the crash of 1987. Notice the huge clustering of the volatility spanning over several months. See the relative quite period in 1985 and the increasing volatility in 1986 and 1987. The marks for year are placed at Jan. 1st.

Fluctuations and correlations are both ingredients which are supposedly known to play an important role in thermodynamic phase transitions.

In 1996, two independent works [7,8] have proposed that critical phenomena would be possible scenarios for describing crashes. More precisely, it has been proposed

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that an economic index $y(t)$ increases as a power law decorated with a log-periodic oscillation, *i.e.*

$$y = A + B\left(\frac{t_c - t}{t_c}\right)^{-m} \times \left[1 + C\sin\left(\omega \ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for } t < t_c \quad (1)$$

where t_c is the crash-time or rupture point, A, B, m, C, ω and ϕ are free parameters. This evolution $y(t)$ is in fact the real part $\Re$ of a power law divergence at $t = t_c$, if $m > 0$, with a complex exponent $m + i\omega$, *i.e.*

$$y \propto \Re\left\{\left(\frac{t_c - t}{t_c}\right)^{-m+i\omega}\right\} \quad (2)$$

The law for $y(t)$ diverges or converges at $t = t_c$ with an exponent $m > 0$ or < 0, respectively, and this evolution is decorated with oscillations converging at the rupture point t_c. This law is similar to that of critical points, and generalizes the situation for cases in which a Discrete Scale Invariance (DSI) [9] is presupposed.

The complex relationship (1) has been proposed elsewhere in order to fit experimental measurements of sound wave rate emissions prior to the rupture of heterogeneous composite stressed up to failure [10]. The same type of complex power law behavior has been also observed as a precursor of the Kobe earthquake in Japan [11]. Such log-periodic corrections have been recently reported [12] in biased diffusion on random lattices [13,14].

In August 1997, we have performed a series of investigations in order to emphasize crash precursors [15]. We have mainly used the closing values of the Dow Jones Industrial Average (DJIA) and the Standard & Poor 500 (S&P500). A law slightly different from equation (1) has been proposed in [16]. A strong indication of a so-called crash event or market rupture point has been numerically discovered by analyzing data up to August 1997. The occurence date was predicted to be in between the end of October 1997 and mid-November 1997 [15]. The crash occurred effectively on October 27th, 1997!

Even though the crash of October 1997 was predicted, the scientific community is actually divided between those who believe in such a crash prediction/interpretation [15,17] and those who believe that crashes are unpredictable events and our Aug. 1997 report was a lucky guess or at best accidental [18]. We discuss a little bit more the predictability problem and findings in this paper.

Moreover, we present a numerical method for visualizing the log-periodic oscillations. This method is applied herein to three different time series of the DJIA: respectively Jan. 1982–Aug. 1987, Jan. 1992–Aug. 1997 and Jan. 1993–Aug. 1998 periods. All series begin on Jan. 1st of the first year and end on Aug. 31th of the last year. These 6 year long periods are illustrated in Figures 2a to 2c.

Table 1.

period	t_c	real t_c
Jan. 1982–Aug. 1987	87.93 ± 0.03	87.79
Jan. 1992–Aug. 1997	97.97 ± 0.09	97.81
Jan. 1993–Aug. 1998	98.99 ± 0.15	?

2 Methodology and data analysis

In references [16,17], we underlined the fact that there are strong physical arguments stipulating that m could be or even should be taken as "universal". We have proposed the universal $m = 0$ value, *i.e.* a logarithmic divergence. The logarithmic divergence of the index y for t close to t_c reads

$$y = A + B\ln\left(\frac{t_c - t}{t_c}\right) \times \left[1 + C\sin\left(\omega \ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for } t < t_c. \quad (3)$$

One should remark that the full period $[t_i, t_f]$ for a meaningful fit should contain the whole euphoric precursor. We have found in [16,17] that the log-divergence is closer to the real signal than the power-law divergence case. Some criticism about equation (3) has been recently formulated by Sornette and Johansen [19], but some reply to their arguments is already found in reference [16,17]. Whence, let us consider herein equation (3) only.

The log-divergence (3) has 6 parameters. In order to find out a rupture point t_c from a data series, we have performed non-linear fits using only the simple log-divergent function

$$y = A + B\ln(t_c - t) \quad \text{for} \quad t < t_c \quad (4)$$

where the condition $B < 0$ is fixed. That function has only 3 parameters and a good estimation of t_c can be obtained. Both Levenberg-Marquardt and Monte-Carlo algorithms [20] have been used for fitting. In Table 1, the best values of these parameters are given for the 3 studied periods. One observes that the estimated t_c points are close to the "black" days for the first two periods. It should be noticed that a rupture point is predicted for the end of 1998.

Assuming that equation (4) is valid, one should remark that

$$\frac{dy}{dt} = \frac{-B}{(t_c - t)} \quad (5)$$

should be found in the daily fluctuation pattern (Fig. 1). This is consistent with the clustering/increasing fluctuations discussed in the Introduction. However, it is not usually obvious to fit an equation as (5). The fits lead to bad results with huge error bars.

A fundamental point which can be raised is the stability of the fitting results using equation (4) when the number of data points varies [21]. Figure 3 presents the predicted rupture point t_c as a function of the position

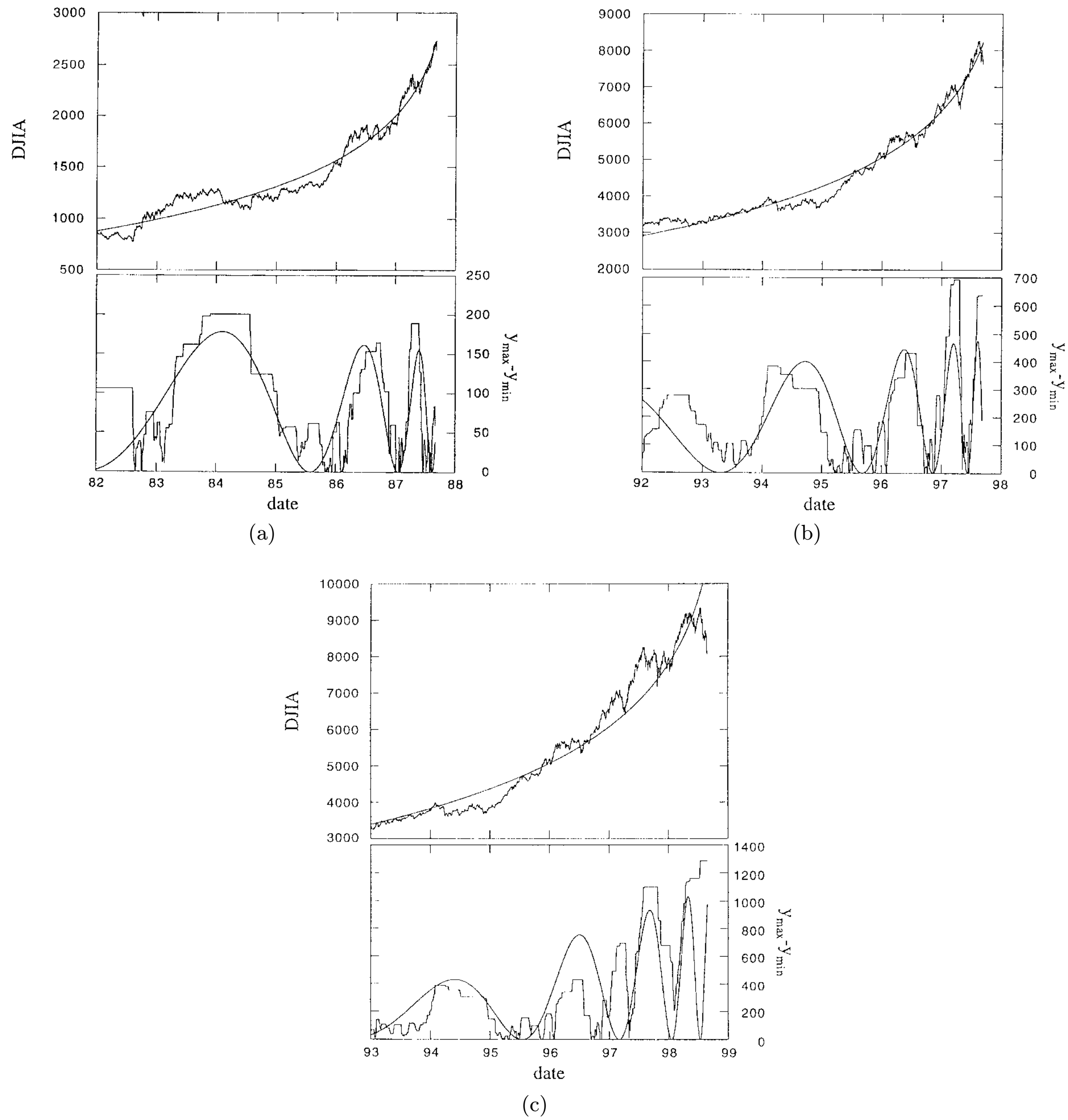

Fig. 2. The DJIA evolution for 4 different periods of euphory, each spanning 6 years: (a) Jan. 1982–Aug. 1987, (b) Jan. 1992–Aug. 1997, (c) Jan. 1993–Aug. 1998. The continuous curves represent the log-divergence fits using equation (4). The pattern at the bottom of each figure represents the envelope of the DJIA which is well fitted by equation (6). The marks for years are placed at Jan. 1st.

t_f of the last data point of the 1982–1987 series. One observes a global convergence of the fits even though there are local bursts moving t_c backwards. The latter changes correspond to downwards fluctuations or corrections of the market. At this point of our investigations, we can conclude that the 3-parameter fit using equation (4) is enough in order to find out the presence of a possible rupture point in such a time series.

Let us now consider the oscillating term of equation (3) which has been quite criticized since no traditional or economical argument so far seems to support the DSI theory at this time. However, the hierarchical structure of the market has been suggested as a possible candidate for generating DSI patterns in [17].

In order to prove that a log-periodic pattern appears before crashes, we have constructed the *envelope* of the index y. Two distinct curves are built: the upper envelope $y_{\max}$ and lower one $y_{\min}$. The former represents the maximum of y in an interval $[t_i, t]$ and the latter is the minimum of y in an interval $[t, t_f]$. At the bottom of

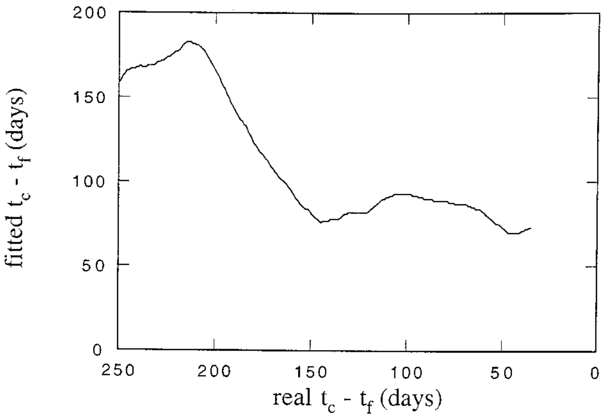

Fig. 3. The evolution of the predicted t_c as a function of the last point t_f of the Jan. 1982–Aug. 1987 data series. Time is given in days and its origin is taken from the true crash day (Black monday, October 19th 1987).

Table 2.

period	ω
Jan. 1982–Aug. 1987	6.68 ± 0.05
Jan. 1992–Aug. 1997	8.91 ± 0.05
Jan. 1993–Aug. 1998	10.26 ± 0.08

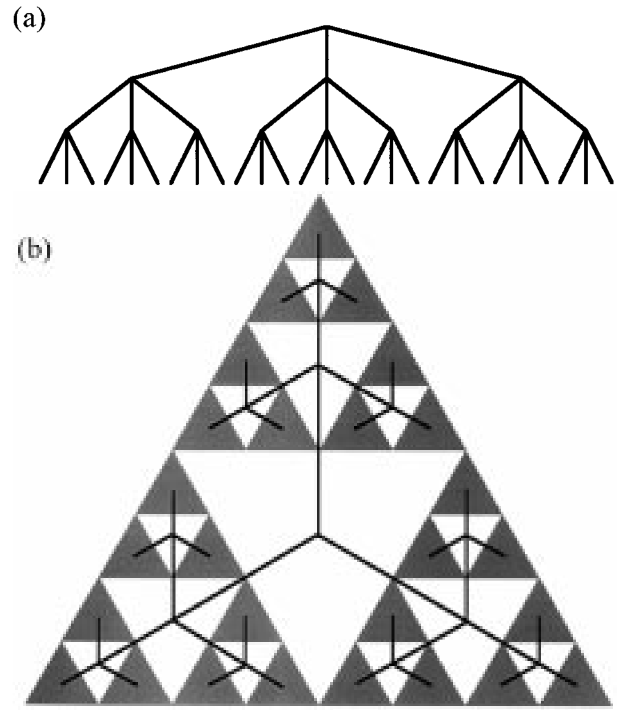

Fig. 4. (a) A shematic representation of a Cayley tree as studied by Amaral and coworkers [22]; (b) another representative of a Cayley tree leading to a discrete fractal representation, *i.e.* a Sierpinski Gasket.

the Figures 2a to 2c, $y_{\max} - y_{\min}$ is presented. One observes a remarkable pattern made of a succession of thin and huge peaks.

When $y_{\max} - y_{\min} = 0$, it means that the index y reaches some value never reached before at a time t and would never have reached if the time axis had been reversed thereafter. This corresponds to time intervals during which the value of the index y reaches new records. In fact, the pattern (see Fig. 2) reflects obviously an oscillatory precursor of the crash. Using the t_c values of Table I, we have fitted the following function

$$y_{\max} - y_{\min} = (C_1 + C_2 t)(1 - \cos(\omega \ln(t_c - t) + \phi)) \quad (6)$$

where C_1 and C_2 are parameters controlling the amplitude of the oscillations. Without these parameters, a fit is still possible if an *a priori* single and constant amplitude (*i.e.* $C_2 = 0$) is presupposed. The above relationship allows us to measure the frequency ω (see Tab. 2). One should remark that for the 1993–1998 period, the oscillatory pattern seems to be accompanied by an additional cyclic term (see bottom of Fig. 2c), probably introduced by the preceeding crash. Some aftershock pattern is known to occur in earthquakes but also in the case of crashes [7] and turbulence also. This should be investigated in future work, and in particular in intra-daily fluctuations.

3 Discussion: Is DSI relevant for stock markets?

We have seen above that a log-periodic pattern exists before crashes. As a consequence, the market should be viewed as a discrete fractal system.

How can we find out such a DSI structure in the stock market? Recently, Amaral *et al.* [22] have studied the statistics of several companies as well as their respective growth. They have found an important result: the growth of companies can be modelled using a hierarchical lattice like a Cayley tree (see Fig. 4a). The fact that the market is hierarchically organized is probably the origin of the log-periodic pattern observed in Section 2. A price model based on such a hierarchical structure has been recently established [23] in order to test this hypothesis.

By analogy and from the above, we conjecture that stock markets are also hierarchical objects where each level has a different weight and a different characteristic time scale (*e.g.* the horizons of the investors). The hierarchical tree might be fractal and its geometry might control the type of criticality [24]. If weights and/or time scales are judiciously chosen, one can build a discrete fractal structure from a Cayley tree (see Fig. 4b). Nevertheless, to our knowledge, no microscopic model is actually able to simulate a crash. This is a challenge left for future works.

4 Conclusion

Three different periods of the Dow Jones Industrial Average (DJIA) have been analyzed. By considering the envelopes of the DJIA, we have demonstrated that before crashes, a log-periodic pattern exists.

Even though error bars are intrinsically large, it is surprising to see that a rupture point for the end of 1998 is easily predicted from data taken before the end of August 1998. The stability of this result should be tested in real-time and on other scales for the best future of economic systems.

5 Additional note

After the completion of this work (in September 1998), dramatic losses have been reported on stock markets at the beginning of October 1998. Among others, the DJIA fell by 4% during the first week of October while the German DAX index fell by 15% during 2 days. This drastic correction of the various stock markets could be probably interpreted as the rupture point predicted in September 1998 (see Tab. 1) when this work was completed. If it is the case, the rupture point is much in advance with respect to the predicted value, *i.e.* $t_c = 98.77$ instead of the predicted $t_c = 98.99$ value from Table 1. This is likely due to the number of data points and to anticipate processes. Nevertheless, the model seems to remain a useful indicator of crashes.

Note added in proof

After a communication from the editor about paper reference [19], and following a request by the referee, we have to comment about one statement in [19] made on a previously published work [16]. The authors of reference [19] consider in their fourth footnote that we have misrepresented their formula equation (1) in [7]. We agree that we misrepresented their formula.

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